Exercise 30

The frequency of vibrations of a vibrating violin string is given by

$$f = \frac{1}{2L}\sqrt{\frac{T}{\rho}}$$

where L is the length of the string, T is its tension, and ρ is its linear density. [See Chapter 11 in D. E. Hall, *Musical Acoustics*, 3rd ed. (Pacific Grove, CA: Brooks/Cole, 2002).]

(a) Find the rate of change of the frequency with respect to

  (i) the length (when T and ρ are constant),

  (ii) the tension (when L and ρ are constant),

  (iii) the linear density (when L and T are constant).

(b) The pitch of a note (how high or low the note sounds) is determined by the frequency f. (The higher the frequency, the higher the pitch.) Use the signs of the derivatives in part (a) to determine what happens to the pitch of a note

  (i) when the effective length of a string is decreased by placing a finger on the string so a shorter portion of the string vibrates,

  (ii) when the tension is increased by turning a tuning peg,

  (iii) when the linear density is increased by switching to another string.

Solution

Part (a)

The rate of change of the frequency with respect to length is

$$\frac{df}{dL} = \frac{d}{dL}\left(\frac{1}{2L}\sqrt{\frac{T}{\rho}}\right) = -\frac{1}{2L^2}\sqrt{\frac{T}{\rho}}.$$

The rate of change of the frequency with respect to tension is

$$\frac{df}{dT} = \frac{d}{dT}\left(\frac{1}{2L}\sqrt{\frac{T}{\rho}}\right) = \frac{1}{2L}\frac{1}{2}\left(\frac{T}{\rho}\right)^{-1/2}\cdot\frac{d}{dT}\left(\frac{T}{\rho}\right) = \frac{1}{4L}\left(\frac{\rho}{T}\right)^{1/2}\cdot\left(\frac{1}{\rho}\right) = \frac{1}{4L\sqrt{T\rho}}.$$

The rate of change of the frequency with respect to linear density is

$$\frac{df}{d\rho} = \frac{d}{d\rho}\left(\frac{1}{2L}\sqrt{\frac{T}{\rho}}\right) = \frac{1}{2L}\frac{1}{2}\left(\frac{T}{\rho}\right)^{-1/2}\cdot\frac{d}{d\rho}\left(\frac{T}{\rho}\right) = \frac{1}{4L}\left(\frac{\rho}{T}\right)^{1/2}\cdot\left(-\frac{T}{\rho^2}\right) = -\frac{1}{4L}\sqrt{\frac{T}{\rho^3}}.$$

Part (b)

If the effective length becomes shorter, then df/dL, the rate of change of the frequency as the length increases, becomes a much bigger negative number. An equivalent, more useful, way to interpret this is that the rate of change of the frequency becomes a much bigger positive number as the length of the string decreases. (The pitch increases.)

If the tension is increased by turning a tuning peg, then df/dT, the rate of change of the frequency as the tension increases, becomes a smaller positive number. (The pitch increases.)

If the linear density is increased by switching to another string, then $df/d\rho$, the rate of change of the frequency as the density increases, becomes a smaller negative number. (The pitch decreases.)